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The Ocularcentric Paradigm in Mathematics: Historical Roots and Contemporary Consequences

Abstract

Although recent decades have observed rapid developments in technology, communication, and educational practice, contemporary culture remains deeply shaped by the so-called ocularcentric paradigm: the assumption that sight is the principal sense and that knowledge is fundamentally derived from visual inspection. This approach is particularly pronounced in mathematics, a discipline commonly associated with demonstrable certainty, but frequently regarded as dependent upon graphical representation of abstract ideas. As a result, mathematical activity is often non-accessible to individuals with low or no sight. This paper seeks to challenge this pervasive misconception. It first traces the historical emergence of ocularcentrism, with particular attention to its entrenchment in Western intellectual traditions. It then examines a selection of notable contributions by blind scientists over the past three centuries, highlighting the generative, structural influence that blindness exerted on their cognitive approaches, research practices, and innovative achievements. By foregrounding these often-overlooked epistemic dynamics, the paper argues for a reconsideration of blindness not as a deficit, but as a potentially productive mode of engaging with mathematical thought.

Keywords: ocularcentrism, disability studies, mathematics education, blind mathematicians, epistemic diversity, visual culture

1. Introduction

Mathematics is commonly understood as the most abstract of disciplines, dealing with universal truths that transcend human experiences. Yet mathematical practice and pedagogy remain profoundly embodied, and specifically, profoundly visual. From ancient Greek geometry's reliance on diagrams through nineteenth-century blackboard culture to contemporary digital visualisations, mathematical knowledge has been systematically presented as something to be *seen*. This ocularcentric paradigm, the assumption that mathematical insight fundamentally requires visual intuition, shapes every level of mathematical culture. Textbooks overflow with diagrams and graphs, pedagogical explanations depend on spatial-visual metaphors, and mathematicians routinely describe their work through the language of sight: "seeing" solutions, achieving "clear" proofs, finding "illuminating" connections. For a discipline ostensibly concerned with abstract, universal truths, mathematics exhibits a remarkably narrow sensory epistemology.

This ocularcentrism has profound consequences. It positions blindness as an epistemic barrier to mathematical work, a deficit requiring heroic compensation or, more commonly, directing blind students away from mathematics entirely. Research consistently shows that blind and partially sighted students abandon mathematical study at disproportionate rates, deterred by visual pedagogies, inadequate teacher preparation, and the pervasive assumption that mathematics is fundamentally inaccessible to them (Bell & Silverman, 2019). The result is systematic exclusion from an entire domain of human knowledge and the professional pathways it enables. Yet if mathematics is truly abstract and universal, why should sensory modality matter? The tension between mathematics' claimed universality and its deeply visual practice reveals something essential about how mathematical knowledge is produced, communicated, and gatekept.

This paper challenges the ocularcentric paradigm by examining what happens when mathematicians work outside its constraints. We argue that the exclusion of blind mathematicians does not merely create barriers to participation, but actively impoverishes mathematical practice by eliminating cognitively diverse modes of reasoning that have historically generated innovative solutions. Through historical analysis and case studies spanning three centuries, we demonstrate that blind mathematicians have not simply succeeded "despite" their blindness through extraordinary individual effort, but have, in several instances, produced contributions that were epistemologically enabled by their non-visual cognitive approaches. Blindness, we argue, can function as an epistemological resource rather than an insurmountable barrier, generating mathematical insights that elude sight-based reasoning.

In making this argument, we build on an emerging body of scholarship that reconceptualises disability not as deficit but as generative difference. Following Bauman and Murray's (2014) influential reframing of deafness as 'Deaf Gain', which foregrounds the unique cognitive, creative, and cultural contributions arising from deaf experience, scholars have increasingly examined 'blindness gain': the distinctive epistemic and creative capacities that blindness enables (Thompson & Warne, 2018; Chottin et al., 2025). To date, this work has been explored primarily within artistic practice and museum studies (Wilder, 2025; Kleege, 2019). The present article extends the blindness gain framework into new territory, demonstrating that blindness functions as an epistemological resource not only in the arts but in mathematics, a domain typically assumed to be either purely abstract, and therefore disability-neutral, or irreducibly visual, and therefore inaccessible to blind practitioners.[1]

2. The Ocularcentric Paradigm

In this section, we explore the historical roots of the ocularcentric paradigm. This historical trajectory reveals a pattern: while ocularcentrism in philosophy has ancient roots, its specific manifestation in mathematical practice evolved through distinct phases. Greek mathematics established visual proof as foundational, medieval and Renaissance thinkers consolidated sight's epistemic primacy, Enlightenment philosophers briefly challenged this hierarchy (enabling the emergence of blind mathematicians), but 19th century mathematical culture paradoxically intensified visual dependence even as mathematics became more abstract. Understanding this history reveals why contemporary mathematics education remains so thoroughly ocularcentric despite ostensibly universal mathematical truths.

Ancient Foundations: Plato, Aristotle, and the Hierarchy of Senses

It is well-known that the first instance of the idea that knowledge comes primarily from sight has to be found in the works of two of the most influential philosophers of ancient Greece. In his *Republic*, Plato defines sight as the eye of the soul and draws a direct comparison between the sun and the eye, both seen as true sources of light and knowledge (Bloomberg, 1993). In the introductory part of his *Metaphysics*, Aristotle describes sight as the noblest of senses. He argued that, unlike touch or hearing, it offers a clear perception of motion, shapes, and colours, representing the most universal means of understanding the universe (Reale, 2000).

Not all ancient philosophers agreed with this hierarchy. Democritus and Lucretius privileged touch, while Hierocles promoted the idea that all senses equally contribute to understanding the world (Kambaskovic & Wolfe, 2014; Ramelli, 2009). Despite these dissenting voices, the Aristotelian idea of sight as the noblest of senses, possibly because of Aristotle's prominent role in the development of the Christian philosophical tradition, became mainstream, particularly in the western world, during late antiquity, the middle ages, and the early modern era.

This ocularcentric approach had a profound effect on the classical mathematical tradition. The recurrent use of the so-called text-to-diagram interaction, the reliance on the visual intuition provided by a drawing, became fundamental to proofs by Greek mathematicians such as Euclid, Archimedes, and Apollonius of Perga (Netz, 1999). The role of visual intuition in *The Elements* does not limit itself to the books where geometry is the main focus, e.g., books I, IV, VI, XI, or XIII, but is equally apparent in those related to topics such as proportions, irrationals, progressions, and number theory. For instance, in book VIII, proportional numbers and compound ratios are introduced via drawings of line segments having visually proportional lengths, whilst book X heavily relies on visual representations of rational and irrational numbers, depicted as straight intervals of incommensurable lengths (Fitzpatrick, 2008). Notably, in Euclid's work, sketches and schemes are not just visual supports, but educational instruments for deduction, logical inference, and the comprehension of abstract concepts (Netz, 1999). In his pioneering work *The Sandreckoner*, Archimedes makes extensive use of the text-to-diagram method to anchor enormous quantities in a visual form, allowing for an intuitive educational representation of myriads, orders, and the hierarchy of myriads (Heath, 1897). Greek mathematics was solidly founded on a juxtaposed conjunction of prose and figures, two factors having equal importance in deduction processes. Thus, given the pivotal role of ancient Greece in the development of mathematics, the ocularcentric paradigm, and its consequences such as the text-to-diagram interaction, became central to the concept of scientific truth and of mathematical proof itself.

Medieval Consolidation

While Greek mathematical practice established visual proof as normative, medieval thinkers provided the philosophical justification that would entrench ocularcentrism within Christian intellectual culture. Among the most illustrious medieval voices who echoed the axiom of a hierarchy of senses, embedding it in a Christian framework, there are Thomas Aquinas, who privileged sight and hearing above all others, as he argued that the eye and the ear are used for rational intents, whilst the organs of touch, taste, and smell only serve more trivial needs (Eco, 1986); and mathematician and physicist Robert Grosseteste, who affirmed the central role of light and physical observation in the pursuit of truth (Lewis, 2013). The Aristotelian stance on the subject also influenced Islamic thinkers like Ibn Sina (Avicenna), who regarded sight as the most precise sense, the one connecting the external and the internal perception (McGinnis, 2010), though ocularcentrism is mostly associated with western culture.

This consolidation set the stage for Renaissance thinkers to extend ocularcentrism from metaphysics into empirical methodology, transforming how natural philosophers conceived the very pursuit of knowledge.

Renaissance Mathematization of Nature

The Renaissance heavily reinforced the Aristotelian sight-driven model of knowledge. 16th and early 17th century thinkers advanced the vision known as the mathematization of nature, namely the idea that the world could be analysed merely through the results of observation and measurable quantities, a line of thought that consolidated ocularcentrism in science (Van Dyck, 2022). Among the voices that promoted this line of thought is Galileo Galilei, whose works in mathematics and astrophysics greatly reinforced the idea of a hierarchy of senses in which sight had clear predominance. He analyses the intrinsic structures of touch and taste, proposing a mechanistic model to reduce them to primary qualities, but still remarks that sight is the noblest of senses because of its natural connection to light (Kambaskovic & Wolfe, 2014; Piccolino & Wade, 2007).

Moreover, the Renaissance saw the emergence of a new algebraic notation, but it remained tied to spatial reasoning, with symbols interpreted through geometric diagrams, proportional line intervals, and visual layouts rather than fully abstract, symbolic operations (Gavagna, 2014; Giusti, 1992). Thus, even as mathematical notation evolved towards symbolic abstraction, it remained anchored in spatial, visual representation, a pattern that would persist even through later revolutionary abstractions. The Christian Middle Ages and the Renaissance had, thus, thoroughly entrenched the ocularcentric paradigm. Yet the Scientific Revolution would bring unexpected challenges to this consensus.

Enlightenment Challenges and the Emergence of Blind Mathematicians

The scientific revolution, and in particular the renewed attention to the study of light and colour, brought many scholars to reconsider these study objects, moving away from the idea of them as merely visual phenomena to promote a more touch-based or touch-influenced approach to the research of knowledge, thus also modifying the well-established hierarchy of the senses (Paterson, 2016). While René Descartes privileged sight but removed its mystical associations, scholars like

William Charlton, Johannes Kepler, Claude Le Cat, and Robert Hooke all diverged from the classical Aristotelian conception to promote a more integrated model of perception. In a letter to Galileo Galilei, Kepler states that Giordano Bruno was superior to both of them because he could understand the motion of celestial objects without even observing them, a powerful testament to a new way of conceiving science as a non-strictly visual inquiry (Kambaskovic & Wolfe, 2014). Le Cat and others argued that light must have a sort of material, almost tangible form, and that therefore sight reduces to a different tactile experience (Kambaskovic & Wolfe, 2014). Echoing Democritus' thought, Le Cat even claimed that, despite its key role in science, sight could be misleading and therefore needed the corrective support of touch (Le Cat, 1750).

It is perhaps not coincidental that the late 17th and the 18th century saw the emergence of the earliest known blind mathematicians in history, namely Nicholas Saunderson (1682-1739) and Leonhard Euler (1707-1783) (Kalinger, 2019; Mele & Sicignano, 2025). Despite these philosophical shifts, this age of astounding scientific, technological, and philosophical advancement was labelled the "Enlightenment" - a term that strongly recalls visual imagery, a sign that the new integrated approach had not completely replaced the classical ocularcentric paradigm. This brief philosophical opening, however, proved fleeting.

Nineteenth-Century Retrenchment and the Paradox of Visual Abstraction

As the materialistic and rationalist philosophical schools became less fashionable, the Aristotelian perspective became dominant again in the 19th century, though sometimes modernised and nuanced. George Wilhelm Hegel placed sight above all other senses, reaffirming its universality and objectivity, embodying his considerations in a more spiritual framework (Houlgate, 1993). Furthermore, Hermann Von Helmholtz and his disciples claimed that scientific knowledge was strictly related to sight, as this sense provided the clearest and most reliable way to analyse the world and to bridge physiology, mathematics, physics, and other sciences (Von Helmholtz, 1925).

The 19th century constituted a period of pivotal mathematical revolutions, including the arithmetization of analysis, the foundation of topology, the first steps of modern algebra and group theory, the Erlangen program, and the birth of non-Euclidean geometries (Neumann, 1999; Boyer, 1991; Boyer, 1959; Houzel, 2005; Drago, 2011). All these crucial moments provided decisive changes of direction for the main branches of mathematics towards abstraction. Paradoxically, this abstractive turn did not diminish visual culture in mathematics but intensified it. It is in this period that blackboard lectures became the standard pedagogical format, transforming mathematical communication into a fundamentally visual performance (Phillip, 2017; Targhetta, 2015). Mathematicians increasingly expected students and colleagues to 'see' proofs develop spatially on the board, internalizing visual-spatial reasoning as the privileged mode of mathematical thought.

This paradox is nowhere more evident than in Felix Klein's groundbreaking 1872 Erlangen program, which promoted a new, more abstract, and unified conception of geometry based on the key notion of invariance, yet presented this abstraction in an overwhelmingly visual framework (Rowe, 2025). Even topology, founded on concepts like connectivity and continuity that transcend visual-spatial intuition, developed a pedagogical culture saturated with diagrams of knots, surfaces, and manifolds. Abstract algebra emerged as a field concerned with purely symbolic operations on groups, rings, and fields, yet Klein's geometric approach to group theory dominated pedagogy. Visual metaphors pervaded mathematical language itself: mathematicians spoke of 'seeing' solutions, achieving 'clear' proofs, and finding 'illuminating' insights. Thus, mathematics became more abstract in content while simultaneously more ocularcentric in practice, a duality that would profoundly shape 20th century mathematical culture and establish patterns that persist in contemporary mathematics education.

Contemporary Mathematics Education and the Persistence of Ocularcentrism

This 19th century consolidation of visual mathematical culture established patterns that persist today. Society is still strongly shaped by an ocularcentric paradigm that, emphasizing the role of sight in the pursuit of truth and knowledge, underestimates other forms of perception. Every aspect, from technology to media, from online contents to transports, is prevalently designed upon the needs of a majority of sighted individuals; consequently, just as everyday vocabulary (San Roque et al., 2015), even science and science education feel the effect of the hegemony of sight (Paterson, 2016). Among the results of this approach is a persistent prejudice that penalises people with low or no sight, often preventing them from gaining access to entire branches of culture and, therefore, to a number of professional paths.

One of the most common stigmas regards blind people's capability in mathematics, a misconception that discourages many blind students from including mathematics in their curricula (Maguvhe, 2015; Francis & Maguvhe, 2008; Bell & Silverman, 2019; Hayes & Proulx, 2023). Historically, mathematics, as the subject mostly associated with reasoning, deduction, and the research of demonstrable truths, has always been perceived as a visual subject. Even pioneer mathematicians like Felix Klein and Waclaw Sierpinski involuntarily stressed this aspect in their seminal pedagogical works. In fact, even the latter, a renowned exponent of the highly-formal Polish mathematical school, often relies on visual intuitions to convey key concepts such as openness, connectedness, boundaries, fractals, and continuum (Klein, 1932; Sierpinski, 1952, 1964). This tendency to consider visual representations as central educational tools has been reinforced in recent decades by the very influential works of scholars like Seymour Papert and Alan J. Bishop, who advocated for a visual, exploratory, constructionist, and computer-based approach to the teaching of mathematics (Bishop, 1991; Arcavi, 2003; Papert, 1980).

According to the World Health Organization, about 3% of the world's population is made up of people with low or no sight (WHO, 2016). However, many students who are blind or partially sighted, particularly those from countries where schools for the blind are not considered mere support but an inevitable surrogate to standard educational paths, cannot access quality mathematics education. In the vast majority of cases, they complain about a lack of specific preparation from their teachers and the urgent need for more adequate methods, texts, and materials. In fact, as products of the deeply rooted ocularcentric paradigm, mathematics school books are generally designed for children with full sight; they are often based on a visual approach with a recurrent text-to-diagram interaction strategy, i.e., frequent references to figures, sketches, vignettes, and graphs (Emerson & Anderson, 2018; Bayram et al., 2015; Bateman et al., 2018; Pinao et al., 2016). Moreover, many teachers design their lessons on visual, two-dimensional drawings and schemes (Hayes & Proulx, 2023), and completely ignore effective non-sight-based solutions, strategies, and methods. For example, only 10% of teachers are fully aware of the problem of ambiguity in the verbalisation of mathematical formulas and an even smaller percentage of them is familiar with the MathSpeak protocol (Vandana & Singla, 2022; Mele & Sicignano, 2021; Soiffer et al., 2019; Isaacson et al., 2010).

Blind students also lament an even more worrying effect of the ocularcentric paradigm: research evidence shows many of them have lost interest in mathematics because their teachers gave them the impression this subject was not accessible to learners with visual impairments (Bayram et al., 2015; Maguvhe, 2015). On the other hand, teachers often complain about the lack of specific pedagogical preparation to present mathematics to students with low or no sight. This also reflects a lack of motivation and confidence when facing the task of including blind students into the learning process (Hayes & Proulx, 2023; Maguvhe, 2015). Hence, the most tangible result of the ocularcentric approach to mathematics is, as far as education is concerned, that the vast majority of students with low or no sight introject the stigma and/or feel the consequence of an inadequate environment and, thus, abandon the study of mathematics at an early stage, renouncing a fundamental part of their education, a wide variety of professional careers, and an essential instrument to navigate the world (Bell & Silverman, 2019). Consequently, despite the rapid improvement and development of assistive technologies in recent decades, the number of blind or partially sighted students who enrol in mathematics university degree courses and work in STEM is still extremely low (Martin et al., 2011; Yusof et al., 2019). The few professional blind or partially sighted mathematicians of recent decades confirm that the environment they personally encountered

during their school years and university path was at times heavily reliant on visual support, thus non-accessible to them, and that they had to create new and highly personal strategies to overcome the obstacles caused by an exclusionary context (Baggett, 2012; Nemeth, 2006; Castigli, 2024).
It is now evident that the often-non-inclusive way mathematics is presented, both to students and to the public, is firmly rooted in the ocularcentric paradigm. The ancient, Aristotelian-like, and outdated idea that mathematical concepts and truths are naturally related to sight and visual intuition, and that therefore they are not fully accessible to the blind and the partially sighted, is a key factor in the way this subject and its closest relatives (e.g., computing, engineering, physics, etc.) are perceived by schools, researchers, practitioners, developers, and the wider population, including individuals with low or no sight. These latter are on the receiving end of a stigma that derives from this approach. However, a detailed investigation of blind mathematicians in history, their biographies, inventions, results and achievements contradicts the ocularcentric paradigm, providing an opportunity to reshape our approach to mathematics in a more complete, inclusive, and equitable way.

This deeply entrenched ocularcentric paradigm has created systematic barriers for blind and partially sighted individuals in mathematics. Yet a detailed investigation of blind mathematicians throughout history reveals a profound paradox: far from being peripheral figures who succeeded 'despite' their blindness, many made contributions that suggest their non-visual cognitive approaches may have been epistemologically productive. The following section examines this history and argues for a fundamental reconsideration of the relationship between sensory modality and mathematical knowledge.

3. Blindness as Epistemological Resource: Historical Case Studies of Non-Visual Mathematical Innovation

The previous section documented how ocularcentrism became systematically entrenched in mathematical practice from ancient Greek text-to-diagram methods through 19th century blackboard culture to contemporary visual pedagogy. This entrenchment operates on a foundational assumption: that mathematical insight fundamentally requires visual intuition. From this follows the corollary that blindness represents a profound epistemic barrier, a deficit requiring accommodation or heroic compensation. This section challenges that assumption through four case studies spanning three centuries, demonstrating that blind mathematicians have produced contributions epistemologically enabled by their non-visual cognitive approaches.[2] These cases reveal not peripheral figures who succeeded "despite" blindness, but mathematicians whose distinctive modes of reasoning generated innovative solutions that, in specific contexts, proved superior to sight-based approaches. Each case illustrates how non-visual cognition, when supported by appropriate institutional conditions, functions as epistemological resource rather than insurmountable barrier.

Nicholas Saunderson (1682-1739): Mental Computation and Algorithmic Innovation

Nicholas Saunderson is today considered the earliest known blind scientist. He was born and baptised in Thurlstone, a small rural village in South Yorkshire, England, in January 1682, and lost his eyesight at the age of twelve months due to smallpox. However, his father John, a tax collector for the local administration, realised that, despite his condition, his first child deserved at least an introduction to literacy. Thus, he started teaching two-year-old Nicholas the alphabet, guiding the hands of his son on the inscriptions on the tombstones in the graveyard that surrounds St. John the Baptist's parish church in Penistone, a historic town near Thurlstone (Davies, 1740). In the wake of this powerful act of trust, which erased every form of self-commiseration in the child, Nicholas entered the local free school and obtained excellent results. He did not excel only in mathematics, but also in literature, French language, Greek, and Latin, acquiring also good mastery on the flute. His ability with mental computations is attested since his youth, together with the creation of the earliest haptic device to perform arithmetic and geometry, namely Saunderson's tactile table (Colson, 1740;

Mele & Sicignano, 2025). The device consisted of a thin, smooth board into which evenly spaced holes were drilled in a regular grid pattern. Each cell in the grid contained a cluster of holes, and Saunderson used pins of two different sizes, distinguishable by touch, inserted into various positions within each cluster to represent the digits 0 through 9. By running his fingers across the board and reading the pin configurations, he could set up numerical operations, track intermediate results, and perform arithmetic and geometrical calculations entirely through touch. The table functioned as a tactile calculator: a physical interface that translated abstract numerical relationships into spatial, haptic patterns, enabling Saunderson to externalise his mental computations without recourse to written notation.[3]

After a brief period at Attercliffe Academy, an institution he rapidly left because of the very metaphysical way mathematics was presented there, he continued studying mathematics on his own with the support of people who read aloud for him (Davies, 1740). In April 1707 he moved to Cambridge, not to enrol as a student, but to ask for an academic position at one of the most prestigious universities in the world. The University of Cambridge proved inclusive and, focusing on Saunderson's abilities rather than on his condition, they welcomed him as a mathematics and physics teacher, nominating him a fellow of Christ's College (Davies, 1740; Tattersall, 1992). During these years, the then Lucasian professor William Whiston used to read aloud for him, until this latter was suspended from the role after a controversy arising from a theological publication. After more than four years in Cambridge, Saunderson had gained such a high status in the academic world that he was chosen, aged only 29, as Whiston's successor on the Lucasian chair. In the following decades, he became a fellow of the Royal Society of London, a founding member of the Longitude Prize board, and one of the most relevant figures of the British scientific scene of the early 18th century. A tireless workaholic, an excellent teacher, a productive scholar, and an example to many influential alumni such as natural scientist William Arderon, philosopher and early psychologist David

Hartley, physician and medical education pioneer Richard Davies, and inventor John Harrison (De-Lacy, 2024; Arderon, 1722; Hartley, 1749; Tattersall, 1992), he kept his role until his death on 19 April 1739 (Wilson, 1838; Tattersall, 1992).

Saunderson might not have been the most prolific mathematician of his time, but he left pertinent scientific results and a legacy that extends far beyond academic research. His mathematical accomplishments were published posthumously in two different works, *The Elements of Algebra in Ten Books* and *Method of Fluxions*, printed in 1740 and 1751 respectively; they contain his original results alongside the contents of his lectures on algebra, geometry, infinite series, combinatorics, optics, astronomy, and more. Among his novel mathematical findings there is at least one that demonstrates how his condition contributed significantly to his scientific achievements.

In *The Elements of Algebra in Ten Books*, Saunderson presents a new algorithm to compute the greatest common divisor of two integer numbers, expressing it as a linear combination of these integers with rational coefficients. For many centuries, the greatest common divisor (GCD) of two integers had been calculated with an algorithm that heavily relies on a sequence of repeated divisions, presented in book VII of Euclid's *Elements*. This procedure starts with the two numbers and, at each step, replaces the largest of them with the remainder of its division by the smallest one; it repeats these operations until it encounters a division with remainder 0, which signals that the GCD has been found. The Euclidean algorithm notes all the passages in a table that records all divisions, quotients, and remainders. Thus, the Euclidean algorithm follows a simple sequence of steps, but it becomes less practical, especially for a blind individual, when the two numbers are quite large, as the divisions can prove difficult to be computed without writing. This is probably the reason why Saunderson devised a variant of this procedure, replacing the repeated divisions with a sequence of repeated subtractions, a far easier task to be accomplished without visual support.

Imagine needing to find the largest shared building block of two large numbers. The standard method requires long division at every step, which is manageable on paper but extremely difficult to track without writing. Saunderson replaced each division with simple repeated subtractions, akin to counting down rather than dividing... his method was not merely a workaround; it was a structurally richer algorithm that simultaneously revealed information the classical method left hidden.

Saunderson's algorithm starts from the two integers and subtracts the smallest one from the largest one until the result is smaller than the smallest integer, then it replaces the largest integer with the result of the subtraction and iterates the operations. The procedure eventually stops when one of the subtractions gives the subtrahend as its result (Saunderson, 1740). For instance, let us apply Saunderson's method to compute the GCD of 60 and 42. Subtracting 42 from 60 gives 18, so, at the end of the first step, we replace 60 with 18. Now, subtract the smallest number from the biggest, 42-18 equals 24; then, once again, since 24 is larger, subtract 18 from it, obtaining 6. This latter is now the smallest number; subtracting it repeatedly from 18 gives again 6; hence the GCD of 60 and 42 is 6.

Moreover, Saunderson could find the rational coefficients of the linear combination of the two integers that returns their GCD in a simple way. For instance, returning to the previous example, 18 is expressed as 1 time 60 plus -1 time 42; 24 is expressed as 1 time 42 plus -1 time 18, then, substituting, as 2 times 42 plus -1 time 60, and so on, reaching in the end the representation of the GCD 6 as a linear combination of 60 and 42 with coefficients 5 and -7 respectively. Notably, the computation of the coefficient can be carried out without writing or with minimal use of step-by-step notes, something Saunderson could easily do on his tactile table (Saunderson, 1740; Tattersall, 2013).

Saunderson's method extends the Euclidean one because it does not merely record the descending sequence of remainders, but simultaneously preserves, at every stage, the exact algebraic ancestry of each remainder in terms of the original pair. The classical Euclidean algorithm stops at the numerical reductions: it shows how each new number arises from the previous ones, but it does not track how those numbers remain expressible as combinations of the initial integers. Saunderson's procedure adds a second, parallel thread that evolves in perfect synchrony with the numerical steps, ensuring that every subtraction is mirrored by a corresponding subtraction of coefficient pairs. This parallel evolution means that the final non-zero remainder comes already equipped with the precise linear combination that represents the greatest common divisor. Hence, Saunderson extends the Euclidean algorithm from a purely computational device into a more comprehensive and accessible method that reveals the underlying structure of the GCD.

The condition in which Saunderson spent almost his entire life played a significant role in the development of his GCD algorithm. First, he devised this procedure because it is substantially easier to mentally compute a sequence of repeated subtractions than a series of divisions, particularly when dealing with very large numbers with four or more digits. Modern research in cognitive sciences and neurosciences confirms that subtraction is by far an easier task to perform mentally than division (Lee-Rosenberg et al., 2011; Shaki & Fischer, 2017). Second, because Saunderson did not rely on the visual representation of the Euclidean sequence, the tabular format presented in Euclid's *Elements*, he was free to design a new and more comprehensive algorithm that not only produces the same results as the classical Euclidean method, but also provides the rational coefficients for the linear combination, all with minimal or no need for visual representation or written notes. Blindness was not a barrier to this innovation, but rather an epistemologically productive factor that generated a solution with distinctive mathematical virtues: computational efficiency, structural completeness, and enhanced accessibility.

Leonhard Euler (1707-1783): From Visual Observation to Structural Abstraction

In the same days Nicholas Saunderson moved permanently to Cambridge, and precisely on 15 April 1707, Leonhard Euler, one of the most prolific and influential mathematicians of all time, was born in Basel, Switzerland. After obtaining a PhD from the University of Basel at the age of 19, he left his native country to work at the Imperial Academy of Sciences of St. Petersburg. The then capital of the Russian empire was his home for two long periods, from 1727 to 1741 and later from 1766 to his death on 18 September 1783; between these two periods he worked at the Royal Academy of

Prussia in Berlin for twenty-five years (Wilson, 1838). He left over 860 original publications in many fields including algebra, combinatorics, number theory, astronomy, calculus, complex numbers, and more; the vast majority produced after he suddenly lost his sight from one eye in 1735 (Dunham, 1999). We do not know the exact reason for this traumatic event as many hypotheses have been proposed, but we know that, probably due to the conjunction of sympathetic ophthalmia and a cataract, he rapidly lost his sight even from the other eye, becoming effectively blind in the following years. In fact, in 1741, he could not recognize characters written on paper and later in 1759 he could not distinguish between a blank page and one with a text written on it (Kalinger, 2019).

In 1736, Euler visited the then Prussian city of Königsberg, now Kaliningrad, Russia, and provided for the first time proof of the abstraction abilities that, within an inclusive context, the mind of a visually impaired or blind person can achieve. The river Pregel bisects the city and its course is further articulated by two distinct islands. Historically, seven bridges bypassed the river: both islands are connected to both banks, two bridges to each side for the bigger island, one bridge to each bank for the smaller one, and the two islands are connected by a seventh bridge. Locals were in search of a path that, crossing each bridge exactly once, started and ended on the same bank; the problem was then known as 'the seven bridges problem'. The task was presented to Euler during his visit. His solution is considered the foundational moment of a very useful and versatile branch of combinatorics known as graph theory (Sachs et al., 1988; Alexanderson, 2006).

Many people, including sighted peers, tried to find a solution to the problem, but it was Euler who proved that such a path did not exist, identifying the conditions under which it would have existed. Whilst others who could rely on the sense of sight concentrated on the specific shape of the bridges and of the river banks, Euler, who could not see the physical instance of the problem in its entirety, internalized the combinatorial core of the bridges' layout and devised a general model that could be applied to many more instances of the issue of finding such a path. He identified land masses with vertices and bridges with edges that could be traversed in both directions, and defined the concept of degree of a vertex, i.e., the number of edges entering/exiting it. Euler's breakthrough lay not in solving the puzzle itself but in recognising that the physical details, the shape of the riverbanks, the length of the bridges, the geography of the islands, were irrelevant. What mattered was solely the pattern of connections. This insight founded the mathematics of networks; the same principles now used to analyse everything from social media connections to transport systems to the structure of the internet.

Euler detached from the concreteness of the real instance to create the first example of a graph, or to be precise of a multigraph, since there are two couples of vertices connected by more than a single edge. This way of modelling the problem allowed him to provide a complete proof of the non-existence of the desired path.He also demonstrated that, for a generic graph, the required solution, now known as a Eulerian circuit, exists if, and only if, every vertex has even degree (Kalinger, 2019; Gribkovskaia et al., 2007; Dunham, 1999). Far from hindering his mathematical abilities, his condition reinforced his shift from concrete visualisation to abstract structural analysis. By relying less on direct physical inspection of the instance and more on symbolic representation, he founded his reasoning on key principles of incidence, adjacency, and connectivity, solving a long-standing problem and giving birth to a new branch of mathematics. Notably, this abstraction was facilitated by the reduction of his dependence on visual exploration.

As his sight deteriorated, Euler became more productive; in fact, a total of 415 papers, more than half of Euler's mathematical output, come from the period after he became completely blind (Kalinger, 2019). He needed the support of sighted assistants to write letters, documents, and papers, but his scientific fecundity was not compromised by his condition. It surprisingly grew, leading to some of his most famous achievements. Among these is his pivotal 1772 work *Theoria Motus Lunae*, which displays a notable example of the overlooked positive effect that, given an inclusive environment, blindness can have on mathematical reasoning (Wilson, 1838; Kalinger, 2019). Put simply, Euler replaced observation with equation. Rather than repeatedly looking through a telescope and plotting the moon's position night after night, essentially assembling an album of visual snapshots, he wrote a set of mathematical equations that captured the moon's entire motion as a unified system, a single formula capable of generating any snapshot on demand.

This work is the earliest systematic and successful attempt to move away from the then standardised, highly visual modus operandi for defining the motion of celestial objects. Instead of relying on orbits modelled after frequent astronomical observations, Euler designed a set of differential equations that allowed him to model the so-called 'three bodies problem', the problem of determining the motion of the moon as influenced by the sun and the earth, in a compact, non-visual, and analytic way (Euler, 2014). Although relatively intricate, these mathematical objects proved to be the right strategy, not only because they were easier to mentally develop, visualise and manipulate for a blind individual, but also because they provided a much more effective algorithm for calculating the moon's position at a given time, a vital resource for sailors and travellers to determine longitude in the following centuries (Wilson, 1838; Kalinger, 2019). The precise determination of the moon's orbit and its practical consequences represent the result of a successful shift from visual recording to mental, mathematical modelling, a change of paradigm that was a direct consequence of Euler's condition and, therefore, of his distinctive approach to the subject.

Twentieth-Century Geometry and Non-Visual Spatial Reasoning

By the early 20th century, geometry had been reconceptualized as the study of invariants under transformations rather than the measurement of fixed figures. This shift towards abstraction might have reduced dependence on visual intuition, yet as Section 2 documented, pedagogical practice moved in the opposite direction: blackboard culture intensified, textbooks became saturated with diagrams, and geometric intuition remained firmly anchored in visual imagery. This created a paradoxical situation for blind mathematicians: while pedagogical barriers intensified, the mathematical objects themselves, topological spaces, manifolds, transformation groups, possessed essential properties that transcended visual representation.

Perhaps consequently, geometry and topology became domains where accomplished blind mathematicians particularly excelled in the 20th century. Other branches, particularly mathematical analysis, became increasingly reliant on manipulating layered equations with extensive symbolic notation, sometimes incorporating characters beyond the standard Braille set. Geometry, despite its intensely visual pedagogical culture, offered distinct cognitive advantages for non-visual reasoning: mental manipulation of spatial transformations, conceptualization of manifolds through local coordinate patches, and analysis of topological invariants could be approached through haptic models, algebraic formalization, and abstract structural reasoning rather than visual diagrams.

Recent neuroscience research provides evidence for distinctive spatial reasoning capacities in blind individuals. Studies demonstrate that blind mathematicians, when engaged in mathematical reasoning, activate not only regions typically associated with mathematical cognition but also the occipital cortex, normally dedicated to visual processing in sighted individuals (Amalric et al., 2018). This neuroplasticity suggests the visual cortex can be repurposed for abstract spatial thinking. Research further shows that blind individuals can develop spatial modelling, orientation, and reasoning abilities that exceed those of sighted individuals, creating novel, non-graphical strategies for mental representations of geometric entities (Shafique et al., 2024; Sierpinska, 2003; Jacobson, 1998). Non-visual spatial cognition is not inferior but qualitatively different, potentially offering advantages for certain geometric and topological problems.

These capacities manifested in achievements of several prominent 20th century mathematicians. Lev Pontryagin (1908-1988), blinded at age 14, made foundational contributions to algebraic topology including Pontryagin duality and cobordism theory, working extensively with abstract topological groups and homology theories. Louis Antoine (1888-1971), blinded in World War I, produced significant results in topology including Antoine's necklace, a pathological embedding demonstrating wild Cantor sets in three-dimensional space. Lawrence Baggett (b. 1939) contributed extensively to harmonic analysis and wavelet theory, working with abstract function spaces, topological groups, and representations. Emmanuel Giroux (b. 1961) has made important contributions to contact topology and symplectic geometry, fields requiring sophisticated understanding of differential manifolds and geometric structures. While these mathematicians' work merits detailed individual examination, their collective presence demonstrates a pattern: under supportive institutional conditions, access to readers, colleagues willing to describe diagrams verbally, departments valuing intellectual contribution over conformity to visual methods; blind mathematicians have repeatedly excelled in fields requiring sophisticated spatial reasoning, challenging assumptions about the necessary relationship between vision and geometric thinking.

The most striking illustration of non-visual geometric reasoning's epistemological productivity comes from Bernard Morin, whose solution to the sphere eversion problem exemplifies how blindness can enable topological insights that elude visual intuition.

Bernard Morin (1931-2018): Topological Thinking Beyond Visual Constraint

Born in Shanghai to French parents in 1931, Morin lost his sight due to glaucoma at the age of 6 and then moved permanently to France. Given his condition, his father directed him towards an academic career in philosophy, but, after a brief period in which he followed the advice, he obtained a degree and a PhD in mathematics, later becoming professor at the University of Strasbourg. During the mid-1950s, he started his research activity under the supervision of Henri Cartan, focusing on the topological problem known as 'the eversion of the sphere' (Jackson, 2002).

Since the mid-20th century, mathematicians had sought a regular homotopy to smoothly swap the two faces of a sphere: a transformation that, allowing self-intersection but forbidding creases, holes, and self-tangencies, turned a spherical surface inside out. In 1957, American topologist Stephen Smale proved that such a regular homotopy existed, but he could not provide a constructive, visual description of the transformation (Smale, 1959). The proof was purely existential; it guaranteed a solution existed without showing how to construct one. It was only in 1967 that the first complete, descriptive solution to the eversion of the sphere was presented by Morin at a conference in Seattle with the use of small clay models he had handcrafted himself, a result later formalized and published in the official gazette of the Academy of Sciences of Paris (Morin, 1967; Sullivan, 1999). Imagine turning a rubber glove completely inside out, but under strict rules. The material can pass through itself, as though ghostly, but it must never be creased, torn, or pinched to a sharp point at any stage. Mathematicians had proved that this was possible for a sphere but could not describe how to actually do it.

The first stage of Morin's construction consists in pushing a point of the sphere towards its antipodal point, e. g. displacing the north pole towards the south pole, until they meet, producing a smooth, tube-like configuration without creases or angular singularities. In the second and most characteristic stage, the tube undergoes a continuous twisting deformation around its symmetry axis, a progressive rotation of the central band of 270°, during which portions of the inner face pass through the surface and generate four protruding lobes. As the twist increases, these lobes move outward and the deformation approaches the halfway configuration: a perfectly symmetric self-intersecting surface with fourfold rotational symmetry, known as Morin's surface. In the third and final stage, the lobes that had been driven inward expand outwards again; the self-intersection simplifies, quadruple points resolve into ordinary double curves, and the remaining small lobes, often called ears, of the original outer face contract. Each ear is bounded by a circle that collapses to a point and disappears as the deformation completes. The resulting surface is once again a topological sphere, but with the original inner and outer faces exchanged (Morin, 1967; Sullivan, 1999). In 1976, Nelson L. Max reproduced the whole eversion process designed by Morin in his celebrated film documentary "Turning a Sphere Inside Out" (Jackson, 2002).

Morin's result demonstrates that his blindness was not an obstacle to spatial modelling, but rather a cognitive advantage. Deprived of visual intuition, he approached the problem of sphere eversion through conceptual, structural reasoning rather than sight-based imagery, bypassing the deceptive constraints that sighted individuals tend to internalize. Sighted mathematicians struggled with the problem precisely because visual intuition suggested that the sphere, as a bounded closed surface, could not be turned inside out without tearing. Visual metaphors of "inside" and "outside" as fixed, impenetrable boundaries actively hindered progress. Morin's reliance on algebraic representation, topological properties understood through symmetry groups and homotopy classes, and tactile models fostered an unusual freedom from these misleading visual assumptions about self-intersection and deformation. This non-visual perspective enabled him to conceptualize the sphere as a malleable manifold governed by symmetries and homotopic properties rather than by rigid, object-like appearances.

In this sense, Morin formulated the first constructive, descriptive solution to the problem not in spite of blindness, but because his mode of reasoning, shaped by his condition, was uniquely suited to internalize and tackle abstract topology problems. Moreover, the analytic beauty of the algebraic equations that describe the solution, the symmetry of the halfway surface, and the fact it uses the minimal number of topological events to accomplish the task make Morin's procedure a solution of recognized elegance (Jackson, 2002; Sullivan et al., 1999). These are not merely aesthetic judgments but mathematical values: elegance in topology often signals deep structural insight, and minimality indicates the solution captures essential rather than contingent features. That a blind mathematician produced what sighted colleagues recognize as an especially elegant solution suggests non-visual approaches can access mathematical truths that visual reasoning obscures.

Zachary Battles (b. 1979): Global Conceptualization and Continuous Functions

The epistemological advantages of non-visual mathematical reasoning extend into contemporary computational mathematics through the work of Zachary Battles, whose reconceptualization of how computers represent functions has transformed numerical analysis. Born blind in Korea in 1979 and adopted by an American family at age four, Battles grew up navigating a mathematical education system designed almost exclusively for sighted students. He pursued undergraduate studies at the University of Pennsylvania before enrolling in the doctoral program at Oxford's Computing Laboratory under the supervision of Lloyd Nicholas Trefethen, himself a leading figure in numerical analysis and matrix computation. This supervisory relationship proved crucial: Trefethen provided not merely technical guidance, but an institutional context that valued innovative thinking over conformity to established visual methods. Battles completed his doctorate in 2005, and the impact of his doctoral work continues to reshape how mathematicians and scientists approach computational problems involving continuous functions.

The problem Battles addressed was fundamental to numerical analysis: how should computers represent and manipulate continuous mathematical functions? The standard approach, dominant since the mid-20th century, treated functions as collections of discrete data points. When a mathematician wanted to work computationally with a function like sin(x) or $\exp(x^2)$, the computer would evaluate the function at a fixed grid of points, for example, 100 or 1000 evenly spaced values, and store these numbers in memory. Any subsequent operation on the function, whether differentiation, integration, or finding roots, would be performed by manipulating this discrete point set. This approach mirrors how sighted mathematicians typically conceptualize functions pedagogically: by plotting them as graphs, collections of (x,y) coordinate pairs that, when sufficiently dense, approximate the continuous curve. The traditional approach treats a mathematical function the way a digital photograph treats an image, as a grid of individual pixels that, collectively, approximate the smooth original. Battles' innovation was akin to replacing this pixel grid with a vector graphic: a single smooth mathematical expression that captures the function perfectly, can be scaled and manipulated without losing information, and reveals the function's inherent properties directly rather than requiring them to be reconstructed from scattered data points.

This point-based paradigm creates multiple computational challenges. Determining how many points are needed for adequate accuracy requires careful analysis and often conservative overestimation. Different operations on the function may require different point densities, necessitating interpolation or resampling. Perhaps most problematically, the discrete representation obscures the function's global properties, its smoothness, its zeros, its extrema, requiring separate algorithms to extract information that is, in principle, inherent to the function itself. The visual metaphor of "plotting" the function thus becomes a computational constraint: computers, like sighted humans sketching graphs, build functions from collections of local samples rather than grasping them as unified mathematical objects.

During his doctoral research, Battles developed a radically different approach, ultimately realized in ChebFun, an open-source software package for MATLAB created in collaboration with Trefethen. Rather than representing functions as point collections, ChebFun represents them as high-degree polynomials using Chebyshev interpolation. The software automatically determines the polynomial degree required to approximate a given function to near machine precision, typically 15-16 decimal digits, on a specified interval. Crucially, this representation is global rather than local: a single polynomial captures the function's behaviour across its entire domain. The polynomial coefficients encode not just function values but the function's complete structure; its derivatives, integrals, and extrema become straightforward algebraic manipulations of these coefficients. Operations that require numerical algorithms in point-based systems become exact algebraic operations on polynomial representations (Battles & Trefethen, 2004).

The technical sophistication of ChebFun lies in its adaptive algorithm: rather than requiring users to specify polynomial degree in advance, the software begins with a low-degree approximation and iteratively refines it, sampling the function at carefully chosen Chebyshev nodes, points distributed to minimize interpolation error. At each iteration, ChebFun tests whether the approximation has converged to machine precision. If not, it doubles the polynomial degree and recomputes. This adaptive approach means users interact with functions as continuous mathematical objects, while ChebFun automatically handles the computational machinery behind the scenes. The result is presented as a vector-like entity in MATLAB, allowing functions to be manipulated with the same algebraic operations used for matrices and vectors, but retaining their essential continuity (Platte & Trefethen, 2010).

Battles’ innovation stemmed directly from his non-visual conceptualization of functions. Where sighted mathematicians and programmers naturally gravitated towards visual metaphors, plotting, graphing, sampling at visible points, Battle approached functions as inherently continuous, global objects best represented algebraically. His blindness necessitated mental manipulation of functions as unified mathematical entities rather than accumulated point collections. This cognitive orientation proved not merely equivalent to visual approaches, but superior for computational purposes: ChebFun routines for differentiation, integration, rootfinding, and solving differential equations often outperform traditional point-based methods in both speed and accuracy. The software has become widely adopted in numerical analysis, with applications ranging from fluid dynamics to quantum mechanics, demonstrating how non-visual mathematical intuition can generate computational innovations with broad practical impact.

Battles’ work exemplifies the contemporary relevance of the patterns identified in earlier case studies. Like Saunderson, he developed a more computationally efficient algorithm by avoiding visual dependence, subtraction replacing division, polynomial representation replacing point sampling. Like Euler, he abstracted from particular instances (individual function plots) to underlying structure (polynomial representation as essential rather than visual manifestation). Like Morin, he reconceived a mathematical object by privileging its intrinsic properties over its visual appearance, functions as continuous algebraic entities rather than accumulated plotted points. And like all three predecessors, he succeeded within an institutional context (Trefethen's Oxford Computing Laboratory) that valued mathematical insight over adherence to established visual paradigms. ChebFun stands as evidence that the epistemological productivity of blindness in mathematics is not merely historical curiosity, but ongoing reality, with implications for how mathematical software, and indeed mathematics itself, might develop if cognitive diversity were recognized as resource rather than deficit.

4. Conclusions: Patterns in Non-Visual Mathematical Cognition

The four case studies examined in this paper reveal consistent patterns in how non-visual cognitive approaches generate distinctive mathematical insights. Across three centuries and multiple mathematical domains, algebra, graph theory, differential equations, topology, and numerical analysis, these mathematicians shared common cognitive strategies that challenge fundamental assumptions about the relationship between sight and mathematical reasoning.

First, all four cases demonstrate a systematic preference for algebraic and structural representation over visual instantiation. Saunderson replaced Euclid's division-dependent algorithm with subtraction sequences more amenable to mental calculation, simultaneously tracking both numerical reductions and their algebraic expression as linear combinations. Euler abstracted the Seven Bridges problem from its concrete spatial layout to the combinatorial structure of vertices and edges, founding graph theory by recognizing that connectivity properties, not geometric appearances, determined the existence of traversable paths. Morin conceptualized sphere eversion through algebraic equations, symmetry groups, and topological properties rather than attempting to trace visual transformations. Battles reconceived functions as continuous algebraic entities represented by polynomial coefficients rather than collections of discrete visual points. In each instance, the mathematician moved away from visual particulars towards underlying mathematical structure, suggesting that reduced dependence on visual representation can facilitate recognition of essential relationships obscured by surface appearances. This is not simply a compensatory strategy, but an epistemological orientation that privileges underlying mathematical structures over visual appearances.

Second, these cases exhibit a pattern of global rather than local conceptualization. Where sighted mathematicians often work from specific visual instances towards general principles, building understanding incrementally from particular examples, these blind mathematicians characteristically began with integrated structural understanding. Euler grasped the Seven Bridges problem in terms of overall connectivity patterns rather than specific bridge configurations, recognizing that the degree sequence of vertices determined traversability regardless of spatial layout. Morin understood sphere eversion through its complete topological properties, the homotopy class, symmetry group, and minimal event structure, rather than attempting to simply trace the transformation step-by-step visually. Battles conceived functions as unified continuous objects characterized by their polynomial approximations rather than accumulations of plotted points sampled locally. This global-first orientation appears to stem from the practical necessity of mentally holding entire mathematical structures simultaneously rather than building them incrementally through visual inspection. Without the ability to glance back at previously constructed portions of a visual representation, blind mathematicians develop cognitive strategies that maintain complete structural awareness throughout the reasoning process.

Third, all four mathematicians developed solutions that were not merely equivalent to visual approaches, but possessed distinctive virtues: computational efficiency, algebraic elegance, structural clarity, or broader applicability. Saunderson's GCD algorithm required less notation and easier mental computation than Euclid's method while simultaneously providing the linear combination coefficients that the Euclidean algorithm left implicit. Euler's graph-theoretic abstraction proved vastly more general than geometric approaches to path-finding problems, enabling a complete characterization applicable to any network structure rather than just the specific Königsberg configuration. Morin's sphere eversion displayed recognized mathematical elegance through its symmetry and minimal topological complexity, using the fewest possible transitions to accomplish the transformation. Battles' ChebFun transformed numerical analysis through its continuous rather than discrete representation, enabling operations that are both faster and more accurate than traditional point-based methods. These were not compensatory workarounds, but genuine mathematical innovations that enriched their respective fields, suggesting that non-visual cognition can access mathematical insights unavailable to or actively obscured by visual reasoning.

Crucially, these achievements required specific enabling conditions. Each mathematician worked within institutional contexts that provided essential support: Cambridge's willingness to appoint Saunderson despite his blindness and provide readers like William Whiston; the Imperial Academy's provision of assistants for Euler who transcribed his dictated work; Cartan's supervision and the University of Strasbourg's departmental support for Morin; Trefethen's collaborative mentorship of Battles within Oxford's Computing Laboratory. Access to assistants, tactile tools (Saunderson's table, Morin's clay models), supportive colleagues who valued verbal mathematical discussion, and most fundamentally, environments that valued mathematical insight over conformity to visual pedagogy proved essential. This underscores that the epistemological productivity of blindness emerges not from individual exceptionalism alone, but from the interaction between cognitive difference and institutional inclusion. Without these enabling conditions, cognitive diversity remains potential rather than actualized resource.

These patterns challenge the ocularcentric paradigm documented in Section 2 at a fundamental level. If mathematical insight can arise from qualitatively different cognitive modes, and if blindness enables rather than merely permits mathematical achievement, then ocularcentrism does not simply create barriers to participation. It actively impoverishes mathematical practice by excluding cognitively diverse approaches. The visual hegemony in mathematics education systematically eliminates precisely those alternative modes of reasoning that have historically generated innovative solutions. When Saunderson's subtraction-based algorithm is dismissed as merely compensatory rather than structurally superior, when Euler's abstraction from visual particulars is attributed to genius rather than cognitive difference, when Morin's topological insight is framed as overcoming disability rather than exemplifying alternative reasoning, when Battles' global function conceptualization is seen as workaround rather than advance; in each case, the mathematical community misses the opportunity to recognize and cultivate cognitive diversity as epistemological resource.

This reconceptualization has profound implications. In demonstrating that blindness functions as an epistemological resource in mathematics, a domain far removed from the artistic and museum contexts where 'blindness gain' has previously been examined (Thompson & Warne, 2018; Chottin et al., 2025), these findings suggest that the generative potential of non-visual cognition may extend across a broader range of intellectual endeavours than current scholarship has recognised. More broadly, our analysis suggests that disability in mathematics is not primarily located in individual bodies, but in the mismatch between diverse cognitive styles and rigid pedagogical paradigms. The problem is not that blind students cannot do mathematics; the problem is that mathematics as currently taught and practiced systematically privileges one sensory modality and excludes cognitive alternatives that have demonstrable epistemic value. Making mathematics accessible requires not merely accommodating blind students within existing visual frameworks, providing Braille textbooks, tactile diagrams, screen readers, but fundamentally rethinking how mathematics is taught, communicated, and conceived.

What would mathematics education look like if it recognized cognitive diversity as epistemic resource? It would teach multiple representations, algebraic, tactile, auditory, kinesthetic, not as accommodations for minority students, but as enriching mathematical understanding for all learners. It would emphasize structural relationships over visual appearances, global understanding over incremental construction from local examples. It would value computational efficiency and algorithmic elegance over conformity to traditional methods. It would recognize that some mathematical insights are more readily accessible through non-visual reasoning and design problems that privilege diverse cognitive approaches. Most fundamentally, it would understand that excluding blind students from mathematics impoverishes not just those students but mathematics itself, eliminating cognitive perspectives that have repeatedly generated innovations benefiting the entire field. The cases examined in this paper demonstrate that three centuries of blind mathematicians have not simply persisted despite an inhospitable environment, but have produced contributions that reveal the epistemological limitations of the ocularcentric paradigm itself. Their achievements challenge us to recognize that the assumed necessity of vision for mathematical work is not a fact about mathematics but a contingent feature of how mathematical culture has developed, and that alternative developments are both possible and potentially superior. Dismantling ocularcentrism in mathematics is, thus, not merely an equity issue, though it certainly is that; it is also a scientific imperative, opening mathematics to cognitive approaches that can advance the field in ways visual reasoning cannot.

Notes:

[1] The concept of 'disability gain' challenges deficit-based understandings of disability by identifying the distinctive epistemic, cultural, and creative contributions that emerge from disabled experience (Barker, 2017). Bauman and Murray's (2014) *Deaf Gain* was foundational in establishing this framework within Deaf Studies. Within blindness studies, 'blindness gain' has been explored in relation to artistic practice: Thompson and Warne (2018) introduced 'blindness arts' in this journal; Chottin, Thompson, and Ware (2025) traced the development from blindness arts to critical blindness studies; and Wilder (2025) examined 'touch-space' and blindness gain in sculptural practice. Garland Thomson's (2024) concept of 'misfitting' offers a complementary resource, proposing that productive friction between non-normative bodies and material-social environments generates distinctive knowledge. Our analysis contributes a new dimension by demonstrating blindness gain within mathematical practice, where the epistemological productivity of non-visual cognition has not previously been systematically examined.

[2] We acknowledge that the mathematicians examined in this article are all men of European origin. This reflects compounded historical exclusions: the barriers that the ocularcentric paradigm created for blind individuals in mathematics were further intensified by the systematic exclusion of women and racialised minorities from mathematical education and professional life. A full intersectional analysis of how disability, gender, race, and geographical location interact to shape access to mathematical practice lies beyond the scope of this article but represents an important direction for future research. For scholarship on gender and mathematics, see Henrion (1997) and Case and Leggett (2005); on race and mathematical achievement, Walker (2014) and Martin (2013); and on non-Western mathematical traditions that challenge Eurocentric historiography, Joseph (2011) remains essential. Contemporary developments suggest this landscape is shifting: the Academy of Blind Scientists, founded in 2025 as the first international academic society led entirely by blind researchers, includes women and scholars from diverse geographical and disciplinary backgrounds, indicating that future studies of blind scientists' epistemic contributions will draw on a more diverse evidence base.

[3] Saunderson's tactile table exemplifies what Dokumaci (2023) terms an 'activist affordance': a creative, improvisational adaptation through which disabled people generate new ways of engaging with environments not designed for them. Dokumaci argues that such innovations are not merely compensatory but constitutively generative, producing tools, techniques, and ways of knowing that would not otherwise exist. Saunderson's device was not an approximation of sighted methods but a qualitatively different interface for mathematical reasoning, one whose haptic logic would later prove essential to his development of the subtraction-based GCD algorithm discussed below. The concept of activist affordance captures a pattern visible across all four case studies in this article: blind mathematicians do not simply work around visual methods but create alternative tools and strategies that generate distinctive mathematical insights.

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